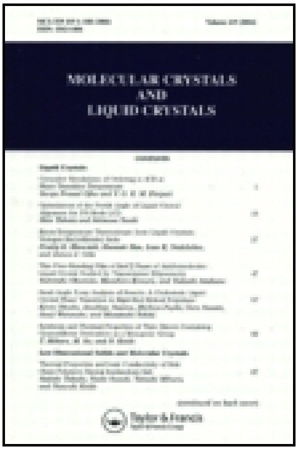

Molecular Crystals and Liquid Crystals
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/gmcl16

Interaction of Liquid Crystals with Electromagnetic Fields, and with Homogeneous and Inhomogeneous Surfaces

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Version of record first published: 21 Mar 2007.

Mol. Cryst. Liq. Cryst., 1987, Vol. 144, pp. 17–32
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Printed in the United States of America

Interaction of Liquid Crystals with Electromagnetic Fields, and with Homogeneous and Inhomogeneous Surfaces†

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(*Received September 15, 1986*)

This review discusses the electromagnetic-field-induced molecular reorientation and bistability in nematic and smectic-C liquid crystals, the interaction of nematic liquid crystals with homogeneous and inhomogeneous surfaces, and the optical properties of spatially inhomogeneous liquid crystal structures.

I. INTRODUCTION

Liquid crystals (LCs) are fluids in which an ordered arrangement of molecules exists. They arise under certain conditions in organic substances with molecules having sharply anisotropic shape, that is, elongated rod-like molecules or flat disk-like molecules. Currently, there are more than 6000 identified liquid crystalline compounds existing over a certain temperature range. A direct consequence of the ordering of anisotropic molecules is the anisotropy of mechanical, electric, magnetic and optical properties. Although LCs combine the properties of an isotropic liquid entirely and a solid partly, they exhibit very specific orientational phenomena, many of which have no corresponding analogues in isotropic liquids or solids. Research on LCs has a history of nearly a century; in the past twenty five years the research effort has been extensive.

†Glenn Brown Award Lecture, presented at the 11th International Liquid Crystal Conference, Berkeley, California, June 30–July 4, 1986

My dissertation on the interaction of LCs with electromagnetic fields and with homogeneous and inhomogeneous surfaces was completed in 1984 under the direction of Professor Robert B. Meyer of the Physics Department, Brandeis University.[1] The research concerned some specific mechanical, electric, magnetic and optical properties. Some of the results and their generalizations are summarized into the following three sections.

In the next section, we summarize the results of a study of electromagnetic-field-induced molecular reorientation and bistability in nematic LCs (NLCs) and smectic-C LCs (SmC).[2-16] In particular, we obtained the exact solution describing the optical-field-induced Freedericksz transition in a homeotropically oriented NLC cell and showed that certain NLCs could have optical-field-induced first-order Freedericksz transitions in which an increasing orienting field results in discontinuous changes in the NLC spatial orientation. Moreover, using an additional dc field, the optical-field-induced first-order Freedericksz transition can always be enhanced or suppressed and hence can be seen in all NLCs at a low laser power. The enhancement of the first-order transition by an additional magnetic field has been confirmed experimentally recently. In addition, we also showed that with an additional optical field, the dc-field-induced Freedericksz transition can become first-order from an otherwise second-order transition, in both planar parallel and homeotropically oriented NLC cells. We also discussed the general properties of the optical-field-induced molecular reorientation in a freely suspended SmC film and gave analytic solutions in the small-distortion linearized regime.

We review in Sec. III the interactions of NLCs with homogeneous and inhomogeneous surfaces.[2,17-24] We showed that variable oblique alignment of NLCs can be achieved on compositionally and microscopically inhomogeneous but macroscopically homogeneous surfaces. The results can be interpreted in terms of a continuum elasticity theory. We also examined the surface effects on the magnetic, electric, and optical field-induced Freedericksz transition in a NLC. The exact solution as well as the general criterion for the transition to be first order were obtained. We also discussed the possibility of surface induced first-order transition and suggested simple experiments in which one might observe the first-order transition.

The last section deals with the optical properties of spatially inhomogeneous LC structures.[25-35] By expressing the electromagnetic fields in terms of antipotentials, the electromagnetic wave in a periodically bent NLC (PBNLC) was shown to have the form of Ince's equation, for which we presented the first reported general solution, and hence the exact solution for the fields. The cases of propagating and totally reflected waves were discussed. We also presented a general method using the geometrical-optics approximation (GOA) for finding the approximate solution for the electromagnetic fields in a layered-inhomogeneous planar anisotropic structure. Explicit expressions for the fields in the zeroth-order and first-higher-order GOA are obtained. The results are applied to a wave propagation in a PBNLC and excellent agreement between the GOA and the exact solution is demonstrated.

II. FIELD-INDUCED MOLECULAR REORIENTATION AND BISTABILITY IN NEMATIC AND SMECTIC-C LIQUID CRYSTALS

A. Optical-Field-Induced Freedericksz Transition and Intrinsic Optical Bistability in NLCs[2,3]

The electromagnetic-field-induced Freedericksz transition (molecular reorientation) in NLCs has been studied extensively in the last twenty five years. But since the first observation made by Freedericksz in 1927, all the observed dc-field-induced Freedericksz transitions in NLCs have been second-order structural transitions in which the change in the NLC spatial orientation is continuous with an increasing orientating field. In the last few years, a great deal of attention has been focused on the optical-field-induced Freedericksz transition in a homeotropically oriented NLC cell.[4] The study of optical nonlinearities via molecular reorientation of LCs is of particular interest because among fluids, LCs have the largest optical-field-induced refractive index changes. The reorientation torque produced by a cw optical field on LCs can result in an extremely strong collective molecular reorientation and large associated nonlinear effects. For example, the light self-focusing nonlinear constant of NLCs has a value larger by eight to ten orders of magnitude than that of carbon disulfide. This study showed that using a high intensity p-polarized normally incident optical field, the electromagnetic-field-induced first-order Freedericksz transition is possible in all NLCs having either planar parallel or homeotropic orientation.

The study of the action of an optical field on a NLC is complicated because the field is propagating in an inhomogeneous anisotropic medium having a dielectric tensor depending on both the intensity of the optical field and the position, and therefore the field and the Poynting vector vary in space. By first showing that the time average of the z component of the Poynting vector is a constant through the medium, the total electromagnetic field energy can then be expressed as the ratio of the intensity of the incident field to the phase velocity, where throughout, the z axis is normal to the NLC cell surfaces, and the xz plane contains the NLC director and the polarization of the incident wave. The Euler equation for the director will then be obtained and consequently the solution describing the orientation of the NLC.[2,3] The solution shows that for a NLC with large optical and elastic anisotropies, the transition can be first order and hence intrinsically optically bistable (OB).[2,3,5] This characteristic distinctly differs from the dc-field-induced Freedericksz transition which is always a second-order transition with rigid surface conditions. Figure 1 shows the maxiumum deformation angle (which is the tilt angle at the midplane of the cell) from the initial homeotropic orientation for the nematics 5CB (4-cyano-4'pentylbiphenyl) and PAA (*p*-azoxyanisole). 5CB shows a second-order transition whereas PAA shows a first-order transition. This form of intrinsic OB does not use a resonant optical cavity and offers the attractive possibility of making large electroresponse devices with a low power sharp threshold.

B. Enhancement and Suppression of Optical Bistability in NLCs[6,7]

Without an external field, the OB criterion depends on the NLCs material parameters: $\kappa_{11}/\kappa_{33} + 9n_0^2/4n_e^2 < 9/4$, where κ_{11} and κ_{33} are the splay and bend elastic constants, n_0 and n_e are the ordinary and extraordinary indices of refraction. By examining data up to a few hundred existing NLCs, we have found that OB could occur in the following three NLCs: PAA, m_3 and m_5.[8] It is of interest to obtain OB from an otherwise second-order transition and also to enhance the width of the bistable cycle. Independently Tabiyan *et al.*,[9,10] and we[6,7] found that with an additional magnetic field or electric field, OB can always be enhanced or suppressed and hence can be seen in all existing nematics. For example, for NLCs with positive diamagnetic anisotropy, the application of a magnetic field parallel to the laser propagation direction (i.e., along the z axis) is a significant control parameter for the enhancement of OB. For an optical-field-induced second-order transition, the transition can become first order for magnetic fields above a critical field, as shown in Fig. 1 for 5CB. For a purely optical-field-induced first-order transition, the bistable width of the hysteresis cycle can be enhanced by the magnetic field, as shown in Fig. 1 for PAA. The enhancement of the first-order transition from a second-order transition by an additional magnetic

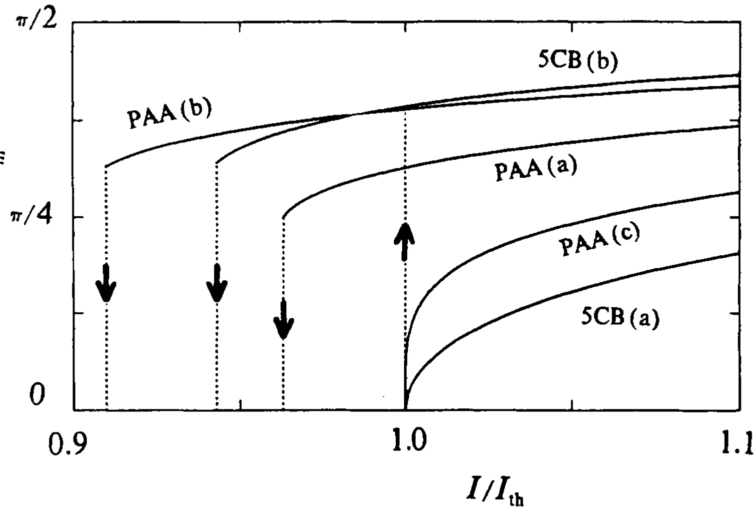

FIGURE 1 Maximum deformation angle θ_m as a function of the reduced intensity I/I_{th} and magnetic field for the optical-field-induced Freedericksz transition in the homeotropically oriented PAA and 5CB cells. We denote $I_0 = (c\kappa_{33}/n_0u)(\pi/d)^2$ and $H_0 = (\pi/d)\sqrt{\kappa_{33}/\chi_a}$; where d is the cell thickness and χ_a is the magnetic susceptibility. For PAA, we put $\kappa_{11} = 9.26 \times 10^{-7}dyne$, $\kappa_{33} = 18.10 \times 10^{-7}dyne$, $n_0 = 1.595$, $n_e = 1.995$. For $d = 250\ \mu m$, $I_0 = 149.0W/cm^2$. (a). $\mathbf{H} = 0$. The transition is first order, i.e., optical bistable with rising threshold $I_{th} = I_0$ and the falling threshold intensity $I'_{th} = 0.96I_0$. (b). $\mathbf{H} = (0,0,0.7H_0)$. The OB cycle is enhanced with rising threshold $I_{th} = 1.49I_0$ and the falling threshold intensity $I'_{th} = 0.91I_{th}$. (c). $\mathbf{H} = (0.6H_0, 0, 0)$. The OB cycle is suppressed and the transition becomes second order with rising threshold $I_{th} = 1.49I_0$. For 5CB, we put $\kappa_{11} = 8.00 \times 10^{-7}dyne$, $\kappa_{33} = 9.51 \times 10^{-7}dyne$, $n_0 = 1.53$, $n_e = 1.73$. For $d = 250\ \mu m$, $I_0 = 60.0W/cm^2$. (a). $\mathbf{H} = 0$. The transition is second order with rising threshold $I_{th} = I_0$. (b). $\mathbf{H} = (0, 0, 2H_0)$. The OB cycle is obtained with rising threshold $I_{th} = 5I_0$ and the falling threshold intensity $I'_{th} = 0.92I_{th}$.

field has been recently confirmed experimentally and provides the first observation of a first-order Freedericksz transition and intrinsic OB in NLCs.[11] The results of both the static and dynamic measurements are in fair agreement with the theoretical predictions, except that the observed hysteresis loops are smoothly varying, indicating possible transverse mode structure. Alternatively, a magnetic field applied transverse to the laser propagation direction (i.e., along the x axis) can suppress the OB from an optical-field-induced first-order Freedericksz transition, as shown in Fig. 1 for PAA.[7]

An additional electric field applied parallel to the laser propagation direction can also be used to enhance or suppress the optical-field-induced first-order Freedericksz transition. The effects depend on the dielectric and optical anisotropies: If $9u/4 > w > 0$, the optical-field-induced first-order transition can be enhanced by the electric field; if $9u/4 < w$ or $w < 0$, the optical-field-induced first-order transition can be suppressed by the electric field; where $\mu = 1 - (n_0/n_e)^2$, $w = 1 - \epsilon_\perp/\epsilon_\parallel$, $\epsilon_\perp$ and $\epsilon_\parallel$ are the dielectric constants perpendicular and parallel to the local NLC director.

C. dc-Field-Induced First-Order Freedericksz Transition in NLCs[12,13]

We also studied external field effects on the dc-field-induced Freedericksz transition in both planar parallel and homeotropically oriented NLC cells. With or without an additional dc field, the dc-field-induced Freedericksz transition is always second order. But with an additional optical field, it is possible to obtain a dc-field-induced first-order transition from an otherwise second-order transition.[12,13]

D. Optical-Field-Induced Molecular Reorientation in a Smectic-C LC[14]

Using a linearly polarized light source with polarization in the plane of the layer at an angle to the initial orientation, it is possible to reorient the azimuthal component of the SmC director, where the z axis is directed in the smectic layer normal. This transition involves only a rotation of the director about the normal to the layers and does not involve any distortion in the layer. Experimentally large reorientations have been observed in a freely suspended SmC film with an incident optical power of less than 50 mW.[15]

We have studied the optical-field-induced molecular reorientation in a freely suspended SmC sample.[14,16] The sample was assumed initially oriented by a homogeneous magnetic field in the smectic-C layer so that without the optical field, the azimuthal angle of the SmC was well aligned in the magnetic field direction. A polarized light beam was then normally incident on the sample. We considered the light polarized at an angle to the magnetic field, so that the azimuthal angle of the director could be varied under the action of the optical field. We have discussed the general properties of the solution for the director and have given analytic solutions in the small-distortion linearized regime. The results showed that the optical alignment effect is localized and does not produce point defects.

Generally, the solutions in the small-distortion regime have the form of zeroth-order Bessel functions. Using the continuity condition at the boundary imposed by the spot size of the optical field, the amplitudes of the deformations can be determined. In regime far away from the optical field, the azimuthal angle approaches its asymptotic orientation exponentially $\sim e^{-qr}/\sqrt{qr}$, where q is the inverse magnetic field coherence length. Threshold power exists if the polarization is normal to the orienting magnetic field. For a magnetic field of the order of 1 *kG*, the threshold power varies from 3 to 120 *mW* for a typical SmC sample with a laser spot size of about 1 to 100 μm in radius. The molecular reorientation can be quantitatively measured by the reflectivity, transmissivity or induced birefringence of a normally incident probe beam. The reflected and transmitted power of the probe beam covering a known area have been derived.

III. INTERACTION OF NLCs WITH HOMOGENEOUS AND INHOMOGENEOUS SURFACES

A. Alignment of NLCs by Inhomogeneous Surfaces[17]

The physics and chemistry of surfaces is becoming more and more important as an exciting field in basic research as well as for devices and technology. Since 1970, studies of LC surface interactions have proceeded actively and several reliable methods for producing structurally inhomogeneous surfaces have been found for achieving the practical result of aligning a LC at a solid surface in some particular orientation.[18] However, in spite of the great effort being devoted to the study of alignment of LCs, the diagnosis and the conditioning of surfaces, and studies of molecular interactions with, and alignments by surfaces, remain among the least understood areas of LC physics. We made a study of the interaction of LCs with compositionally and microscopically inhomogeneous but macroscopically homogeneous surfaces. We presented experimental results showing that the interaction between the LCs and inhomogeneous surfaces leads to oblique alignment and confirms the prediction made by Meyer in 1978.[19] Meyer suggested that controlled inhomogeneous surface allow experimental means to measure the basis parameters describing LC alignment.

The inhomogeneous surfaces are essentially flat and contain some patterns of known materials having different alignment properties for the LC used, resulting in variations over the surface in the local values of the tilt angle and the anchoring strength of the LC director at the surfaces. The competition between the orienting forces in neighboring regions of the sample surface leads to spatially varying structure in the director field at the surfaces, which dies out in the interior of the sample in a boundary layer whose thickness is comparable to the fundamental wavelength of the surface pattern. The resulting equilibrium mean orientation and mean anchoring energy can be controlled by varying the properties of the surface pattern. The systematic production of such surfaces enabled us to control the azimuthal orientation and vary the mean tilt angle of the director continuously from homeotropic to parallel alignment. Moreover, the mean azimuthal orientation of the director could be switched between energetically equivalent orientation directions, an attribute which has potential application to display devices.

Experimentally, we have constructed inhomogeneous surfaces with parallel and homeotropic orientations. Both random and periodic pattern were generated by vapor deposition of metal; random structures of metal islands were made by controlling the thickness of evaporated metal and the periodic structures were made by deposition through a mask. We developed three methods for generating the inhomogeneous surfaces. Here we will discuss one of the methods, in which SiO (silicon monoxide) is used as the parallel alignment agent and the silane compound DMOAP (*N*, *N*-dimethyl-*N*-octadecyl-3-aminopropyltrimethoxysilyl chloride) is used as the homeotropic alignment agent. A 200-Å-thick SiO is first evaporated obliquely at 35° from the surface to give a background parallel alignment. The substrate is next coated with the silane DMOAP. A metal film is then evaporated normally onto the substrate to a thickness appropriate to give metal islands or, evaporating through a mask, to a thickness that gives a continuous film. A low power glow discharge is applied to remove or alter the alkyl chains on the silane-treated surface left unprotected by the metal film to the point that locally the LC alignment would be parallel. We finally removed the metal film by dissolving it in a dilute acid solution.

The resulting substrate should then consist of an inhomogeneous surface with silane patches, typically a few ten of Angstroms in diameter, that favor vertical alignment surrounded by SiO whose local alignment is parallel. The alignment of 6CB (4-cyano-4'hexylibiphenyl) on the inhomogeneous surfaces generated by aluminum as the intermediate metal is shown in Fig. 2. The resulting average tilt angle between the director and the substrate surface initially increases linearly with the aluminum thickness but for 100 to 160 Å, the alignment increases very little. Such a result could be explained by assuming the metal coverage area initially increases linearly and for thickness more than 100 Å the islands grow in height with the effective coverage area remaining unchanged. The alignment depends not only on the ratio of the two different surface areas, but also on the anchoring strengths of the two surfaces. We found also that the alignment is uniform and stable in time.

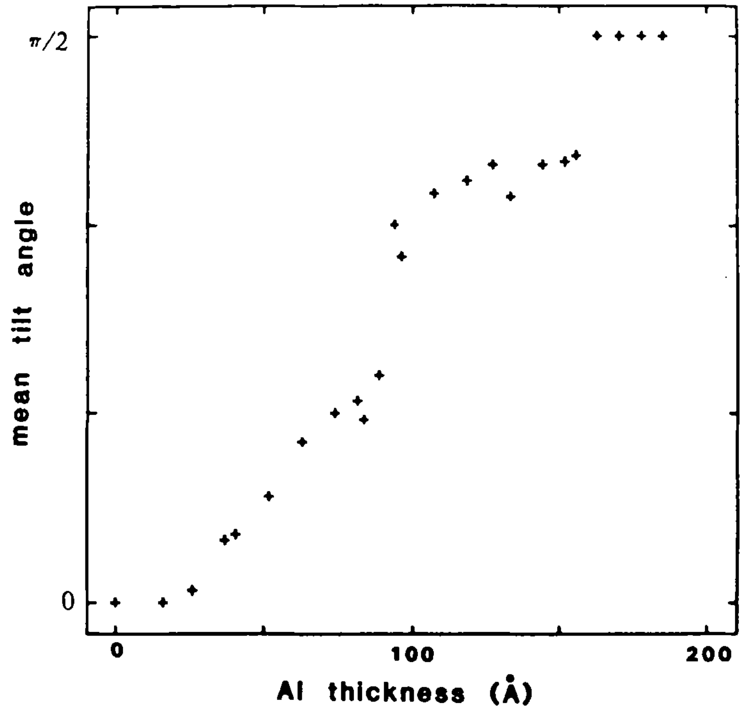

FIGURE 2 Alignment of nematic 6CB by inhomogeneous surfaces generated by method discussed in Sec. IIIA with aluminum as the intermediate metal.

The planar structures lead to an intrinsic twofold azimuthal orientational degeneracy which was confirmed experimentally. A unique orientation of the director can be obtained by heating the NLCs to the isotropic phase and then cooling in an external magnetic field oriented to give a preferred direction. With equal elastic constant and with inhomogeneous surfaces having no intrinsic anisotropy in the azimuthal orientation, the azimuthal orientation always has complete degeneracy. There are two ways to introduce azimuthal anisotropy into the problem. The first is to make the elastic constants unequal, as considered by Berreman[20] and Meyer.[19] The second way is to make one of the homogeneous surfaces have a preferred azimuthal orientation. For example, if an inhomogeneous surface is composed of parallel (or homeotropic) and oblique orientations, then the oblique orientation will have an intrisically preferred azimuthal orientation, which will then determine a preferred azimuthal orientation for the complementary orientation.

B. Multistable NLC Orientation Induced by External Fields and Interfacial Interaction[2,21]

Considerable effects have been devoted to the effects of the interfacial interaction on Freedericksz transition, optical display performance and phase transition. Some experimental findings show that the anchoring strength between NLCs and interfaces are finite and typically of order $1 - 10^{-4}$ *erg/cm.*[2] Evidently the Freedericksz transition's threshold field will be lower as the anchoring strength decreases, and for finite anchoring strength, there exists a finite-valued saturation field, above which the entire NLC sample will orient in the preferred orientation direction determined by the external field.[22,23]

We studied the effects of a homogeneous, short-range, arbitrary strength interfacial potential on the magnetic (MFT), electric (EFT) and optical (OFT) field-induced Freedericksz transition in a NLC and obtained the exact solution. We found that the function $G(\theta_m)$ which gives the field G needed to maintain the NLC state with a given θ_m is a single-valued function of θ_m but is not necessary monotonic, i.e., it can assume the same value for different θ_m, where θ_m denotes the maximum deformation angle in the cell, as compared to its initial orientation. We define the field G to be magnetic field for MFT, applied voltage for EFT, and square root of the incident optical-field intensity for OFT. The function $\theta_m(G)$, which gives the maximum deformation angle at a given maintenance field G, is, then a multi-valued function of G. The transition is second order if G is a monotonic function of θ_m. If the function $G(\theta_m)$ has one or more locally extremal values at some intermediate angles between 0 and $\pi/2$, then several first-order transitions accompanied by hysteresis loops could occur at these angles. We obtained the general criteria for the existence of a first-order transition and showed that surface interactions could induce bistable and multistable states, which open a new possibility for display applications.

For a second-order transition, one can determine the detailed shape of $G(\theta_m)$ using the usual experimental observation methods, such as the induced-birefringence and capacitance measurements. However, for a first-order transition, the usual measurement methods can provide information on the hysteris loops, but not the details of $G(\theta_m)$. Indeed, any physical measurement at thermodynamic equilibrium will be inadequate for revealing details where $dG/d\theta_m < 0$ (first-order transition condition) because these parts of the curve are inaccessible. They might be probed by nonequilibrium measurements, which have not been discussed.

Using the dependence of orientation on the anchoring strength and the cell thickness, three simple experimental methods were suggested to manifest the effects of finite anchoring on the transition. At present, surface-induced first-order transitions have not been observed experimentally. However, there is indirect evidence showing that surface induced bistable and multistable states could occur. We also suggested three simple empirical approaches, for which one may observe the first-order Freedericksz transition induced by the external field and stabilized by the surface interaction. The electric-field-induced and surface-stabilized first-order orientation transitions in ferroelectric smectic-C LCs have been experimentally observed.[24] It is hoped that similar results could be observed in NLCs in the near future. A first-order transition stabilized by surface interactions could have important practical applications.

IV. OPTICAL PROPERTIES OF SPATIALLY INHOMOGENEOUS LIQUID CRYSTAL STRUCTURES

A. Geometrical-Optics Approximation[25,26]

LCs are excellent systems for studying electro-optics, magneto-optics and non-linear optics, since the experimental geometries can be well defined by the appropriate surface conditions and the orientation of the LCs can easily be modified by an external field. The orientational response of the molecules to the external perturbation is a collective effect and is usually extremely large. Because of the surface conditions, however, the induced molecular orientation is generally not uniform across the cell. The study of the electromagnetic wave propagation in a spatially inhomogeneous LC medium is interesting and complicated because the LC medium is an inhomogeneous dielectric medium with large optical anisotropy. The optics of such a medium have generally been studied through numerical methods, such as those developed by Berreman and Scheffer.[27] At present the exact analytic solutions for the fields in inhomogeneous LC structures have been found only for a cholestric LC and for a PBNLC, which is shown in Fig. 3 and which will be discussed in Sec. IVB. Approximate solutions are of significance in the study of wave propagation in inhomogeneous media. They include the GOA, the phase-integral method and the method of perturbation. For slowly varying inhomogeneous isotropic media, the GOA and its extension have been shown to be most suitable. It has been shown by Luneberg and Kline using Duhamel's

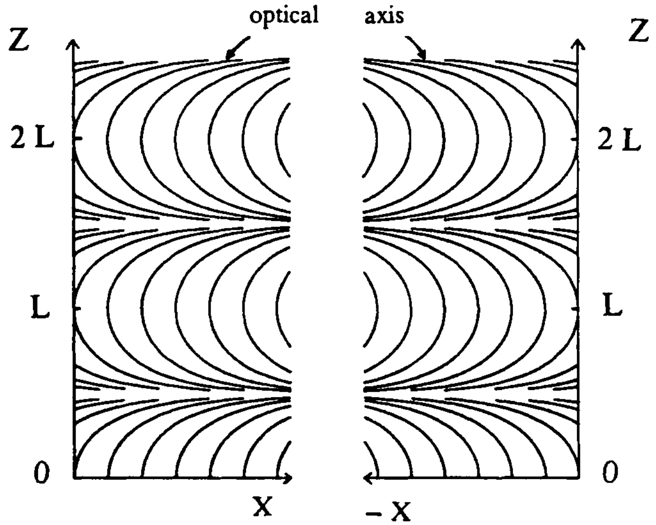

FIGURE 3 The two geometric possibility of a PBNLC.

principle that the GOA is an asymptotic solution of Maxwell's equations.[28,29] For layered-inhomogeneous isotropic media, the GOA is the first approximation to the exact solution for the field.

We have used the GOA approach in order to find approximate solutions for the electromagnetic fields in a layered-inhomogeneous planar LC structure, in which the dielectric properties of the inhomogeneous medium vary very little over distances of the order of the wavelength of the propagating wave. The electromagnetic wave is assumed obliquely incident on the medium with polarization parallel to the plane containing the LC's optical axis, i.e., the xz plane. Consequently only the extraordinary wave is excited in the medium. In the GOA, the phase is almost linear in space and varies much more rapidly than the amplitudes, which are slowly varying functions of space. The advantage of the GOA is that, to zeroth-order, the propagation is described by ordinary differential equations. We obtained the zeroth-order and first-higher-order GOA solutions. By comparing the amplitudes of the zeroth-order and first-order solutions, a validation for the zeroth-order GOA has been obtained. From the integral GOA validation, we also obtained the sufficient condition for GOA to be valid when the optical axis varies monotonically in space. When the condition applies, two waves in the medium propagating in opposite directions are independent of each other, and thus, for the zeroth-order GOA, there is no reflection. In general, the fields and wave propagation direction depend on both the angle of incidence and the azimuthal angle of the incident wave, but the total internal reflection is independent of the two azimuthal angles for the case that we are considering.

The GOA has been used to study the optical-field-induced Freedericksz transition.[2,5,30] As an example for comparison of the GOA to an exact solution, we have considered the electromagnetic wave propagation in a PBNLC.[31] Within the range of expected validity of the GOA, excellent agreement between the GOA and the exact solution is obtained: The difference between the GOA and the exact solution for the fields are less than 2×10^{-5} of the values of the respectively exact solutions, as shown in Fig. 4. The difference between the GOA and the exact solution for the ray propagation direction and the path of the ray are less than 8×10^{-6} of the value of the respective exact solutions.

Using the GOA solutions, we have also shown that for layered-inhomogeneous LC planar structures, at a given frequency, there is some distribution of parameters over the thickness of the layer such

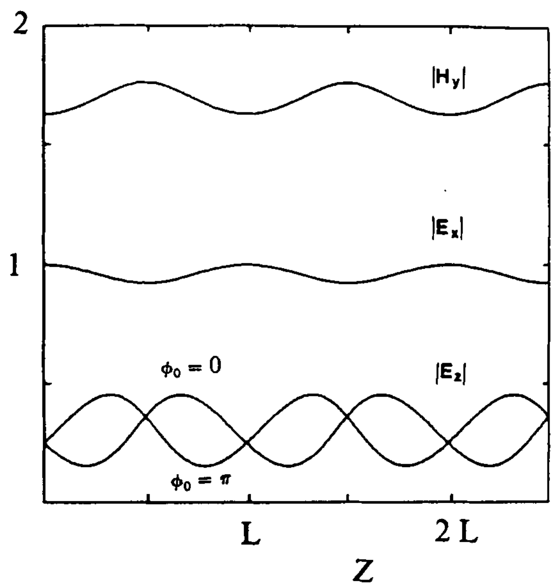

FIGURE 4 The amplitudes of the fields in the GOA and for the exact solution as functions of z for a PBNLC with 5CB as the material. In the calculation, we put $n_0 = 1.562$, $n_e = 1.806$, $\theta_0 = \pi/6$, $\phi_0 = 0$ and $\phi_0 = \pi$; the spatial period of the PBNLC is $L = 20$ μm, and the wavelength of the incident wave is 5145 Å. The amplitudes of the field are normalized so that $|E_x(z = 0)| = 1$

that reflection will be absent for a normally propagating extraordinary wave.[32] A general theorem for constructing the nonreflecting structure in an arbitrary layered-inhomogeneous uniaxial medium was also derived.

B. Electromagnetic Wave Propagation in a PBNLC[31]

Spatially periodically layered media play an important role in a number of applications, such as multilayer high-reflectance coatings, phase gratings with harmonically modulated dielectrics, phase matching structures for nonlinear optical applications, and narrow-band optical bandpass filters. Consequently, considerable effort has been expended by many researchers in analyzing electromagnetic wave propagation in periodic layered media. We studied the extraordinary wave propagation in a PBLNC, in which the optical axis of the uniaxial NLC lies in the xz plane of a Cartesian coordinate system and rotates linearly about the y axis, as shown in Fig. 3. An electromagnetic wave propagating in the positive z direction is obliquely incident on the medium with polarization parallel to the plane of incidence, which is the xz plane. Consequently, only the extraordinary wave is excited in the medium. Since the structure is periodic, wave propagation in this structure is similar to that in solid crystals. In principle, the period of a PBNLC can be on the order of a few hundred angstroms and above. Thus Bragg reflection of light for a PBNLC would occur in the ultraviolet, visible, and infrared regions rather than in the x-ray region as for solid crystals.

To obtain a comparatively simple wave equation for the fields, we expressed the fields in terms of antipotentials[33] and defined a generalized Lorentz gauge condition relating the vector and scale antipotentials. The fields can then be described by Ince's equation,[34] for which a general solution and the normalized solution, and hence the exact solution for the fields, is given for the first time to our knowledge. The method of antipotential used in the study provides a more simple analytic approach, compared to other methods used in the study of the optics of LCs. The equivalence of the field and antipotential descriptions is shown for the case of normal incidence. It is worth mentioning that Ince's equation is important in its own right because it describes a wide variety of systems. Ince's equation is closely related to some known equations with elliptic functions as their periodic coefficients. For example, the Lame equation, the Mathieu equation, the Picard elliptic equation, the frequency modulation equation, and the Hermite elliptic equation can be shown to be special cases of Ince's equation. It was first studied by Magnus and Winkler in connection with the problem of coexistence of two linearly independent periodic solutions for Hill's equations. They have shown that all known cases of Hill's equations with analytic coefficients and with a decidable coexistence problem are special cases of Ince's equation. It is the most-general equation to which Ince's method of a three-term recursion relation can be applied.

We showed that the waves in a PBNLC have the form of Bloch waves and that there exist wavelength bands for wave propagation without attenuation, separated by wavelength bands where waves are attenuated. The fields and the Poynting vector depend not only upon the angle of incidence but also upon the azimuthal angle of the incident wave, as shown in Fig. 4. However, the critical angle for the total internal reflection to occur depends only upon the angle of incidence and is independent of the two azimuthal angles of the incident wave. This conclusion agrees with the GOA result[25,26] and the general theorem[35] that the reflectivity and transmissivity from a layered-inhomogeneous anisotropic structure depend only upon the angle of incidence, but is independent of the two possibilities for the azimuth of the incident wave.

Acknowledgments

I gratefully acknowledge Professor Robert B. Meyer, my thesis advisor, for his guidance, encouragement and support.

As collaborators, friends and teachers, I am grateful to Dr. Alan J. Hurd and Dr. Charles Y. Young, for their continual encouragement and support during my graduate study. I am also grateful to Andy J. Karn, Professors Sergi M. Arakelian and Yuen-Ron Shen, for the collaboration on the optical-field-induced Freedericksz transition in nematics in the last two years.

I benefited from discussions with K. C. Chang, S. Fraden, S. D. Lee, and G. Srajer, Drs. F. Lonberg, D. Andelman, Y. P. Cheung, C. Chiang, S. Garoff, W. E. Howard, R. Lee, V. Taratuta, and K. H. Yang, Professors E. Gross, P. Heller, N. Liang, T. Odagaki and H. Pendleton.

The financial support from Gillette, IBM, Sony, the Martin Fisher School of Physics and the Graduate School of Arts and Sciences of Brandeis University, the U.S. Army Research Office Grant No. DAAG-29-80-K0050, and the National Science Foundation Grant No. DMR-8210477-01, are gratefully acknowledged.

I appreciate the benefits received from Glenn H. Brown's contributions to the field of liquid crystal research, and his interest in young scientists.

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